

# NONLINEAR LONGITUDINAL CURRENT GENERATED BY N TRANSVERSAL ELECTROMAGNETIC WAVES IN THE QUANTUM PLASMA

A. V. Latyshev<sup>1</sup> and V. I. Askerova<sup>2</sup>

*Faculty of Physics and Mathematics,  
Moscow State Regional University, 105005,  
Moscow, Radio str., 10-A*

## Abstract

Quantum plasma with arbitrary degree of degeneration of electronic gas is considered. In plasma  $N$  ( $N > 2$ ) external electromagnetic waves are propagated. It is required to find the response of plasma to these waves. From kinetic Wigner equation for quantum collisionless plasmas distribution function in square-law approach on quantities of vector potentials of  $N$  electric fields is received. The formula for electric current calculation is deduced at arbitrary temperature, i.e. at arbitrary degree of degeneration of electronic gas. It is shown, that the nonlinearity account leads to occurrence of the longitudinal electric current directed along a wave vector. This longitudinal current is orthogonal to the known transversal current received at the linear analysis. The case of small values of wave number is considered. It is shown, that in case of small values of wave numbers the longitudinal current in quantum plasma coincides with a longitudinal current in classical plasma.

**Key words:** quantum plasma, Wigner equation, Fermi–Dirac distribution, longitudinal and transversal electric current, nonlinear analysis.

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<sup>1</sup>avlatyshev@mail.ru

<sup>2</sup>vera\_askerova@mail.ru

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## **1 Introduction**

In the present work formulas for electric current calculation in quantum collisionless Fermi—Dirac plasma are deduced. At the decision of the kinetic Wigner equation describing behavior of plasma, we consider both in expansion of distribution function, and in expansion of Wigner integral the quantities proportional to squares of vector potentials of external electric fields and their products. In such nonlinear approach it appears, that the electric current has two nonzero components. One component of an electric current it is directed along vector potentials of electromagnetic fields. These components of an electric current precisely same, as well as in the linear analysis. It is a "transversal" current. Those, in linear approach we receive known expression of a transversal electric current. The second nonzero an electric current component has the second order of smallness concerning quantities intensity of electric fields. The second electric current component is directed along a wave vector. This current is orthogonal to the first a component. It is a "longitudinal" current. Occurrence of a longitudinal current comes to light the spent nonlinear analysis of interaction of electromagnetic fields with plasma.

Nonlinear effects in plasma are studied already long time [1]–[10].

In works [1] and [6] nonlinear effects in plasma are studied. In work [6] nonlinear current was used, in particular, in probability questions decay processes. We will notice, that in work [2] it is underlined existence of nonlinear current along a wave vector (see the formula (2.9) from [2]).

In experimental work [3] the contribution normal field components in a nonlinear superficial current in a signal of the second harmonic is found out. In works [4, 5] generation of a nonlinear superficial current was studied at interaction of a laser impulse with metal.

Quantum plasma was studied in works [10]–[21]. Collisional quantum plasma has started to be studied in work of Mermin [15]. Then quantum collision plasma was studied in our works [16]–[19]. In works [20] – [21] generating of a longitudinal current by a transversal electromagnetic field in Fermi–Dirac classical and quantum plasma [20] and into degenerate plasma [21]. We will specify in a number of works on plasma, including to the quantum. These are works [11]–[14].

In this paper formulas for electric current calculation into quantum collisionless plasma are deduced at any temperature, at any degree of degeneration of electronic gas.

## 2 The Wigner equation

Let us demonstrate, that in case of the quantum plasma described by kinetic Wigner equation, the longitudinal current is generated, and we will calculate its density. It was specified in existence of this current more half a century ago [2].

Let us turn to the consideration of the fact, that the quantum plasma is in  $N$  external electromagnetic fields with the vector potentials representing running harmonious waves

$$\mathbf{A}_j(\mathbf{r}, t) = \mathbf{A}_{0j} e^{i(\mathbf{k}_j \mathbf{r} - \omega_j t)} \quad (j = 1, 2, \dots, N).$$

Corresponding electric and magnetic fields

$$\mathbf{E}_j = \mathbf{E}_{0j} e^{i(\mathbf{k}_j \mathbf{r} - \omega_j t)}, \quad \mathbf{H}_j = \mathbf{H}_{0j} e^{i(\mathbf{k}_j \mathbf{r} - \omega_j t)}, \quad (j = 1, 2, \dots, N)$$

are connected with vector potentials equalities

$$\mathbf{E}_j = -\frac{1}{c} \frac{\partial \mathbf{A}_j}{\partial t} = \frac{i\omega_j}{c} \mathbf{A}_j, \quad \mathbf{H}_j = \text{rot } \mathbf{A}_j \quad (j = 1, 2, \dots, N).$$

It is necessary to specify, that vector potential of an electromagnetic field  $\mathbf{A}_j(\mathbf{r}, t)$  is orthogonal to a wave vector  $\mathbf{k}_j$ , t.e.

$$\mathbf{k}_j \cdot \mathbf{A}_j(\mathbf{r}, t) = 0, \quad j = 1, 2, \dots, N.$$

It means that the wave vector  $\mathbf{k}_j$  is orthogonal to electric and magnetic fields

$$\mathbf{k}_j \cdot \mathbf{E}_j(\mathbf{r}, t) = \mathbf{k}_j \cdot \mathbf{H}_j(\mathbf{r}, t) = 0, \quad j = 1, 2, \dots, N.$$

We take the Wigner equation describing behavior of quantum collisionless plasma

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + W[f] = 0 \quad (1.1)$$

with nonlinear integral of Wigner

$$\begin{aligned} W[f] = & \mathbf{p} \sum_{j=1}^N \frac{ie\mathbf{A}_j}{m\hbar c} \left[ f\left(\mathbf{r}, \mathbf{p} + \frac{\hbar\mathbf{k}_j}{2}, t\right) - f\left(\mathbf{r}, \mathbf{p} - \frac{\hbar\mathbf{k}_j}{2}, t\right) \right] - \\ & - \frac{ie^2}{2mc^2\hbar} \sum_{j=1}^N \mathbf{A}_j^2 \left[ f\left(\mathbf{r}, \mathbf{p} + \hbar\mathbf{k}_j, t\right) - f\left(\mathbf{r}, \mathbf{p} - \hbar\mathbf{k}_j, t\right) \right] - \\ & - \frac{2ie^2}{2mc^2\hbar} \sum_{\substack{s,j=1 \\ j < s}}^N \mathbf{A}_s \mathbf{A}_j \left[ f\left(\mathbf{r}, \mathbf{p} + \hbar \frac{\mathbf{k}_s + \mathbf{k}_j}{2}, t\right) - f\left(\mathbf{r}, \mathbf{p} - \hbar \frac{\mathbf{k}_s + \mathbf{k}_j}{2}, t\right) \right]. \end{aligned}$$

This integral of Wigner is constructed by analogy to integral of Wigner deduced in work [10] with use of one vector potential.

In the equation (1.1)  $f$  is the analogue of quantum function of distribution electrons plasmas (so-called function of Wigner),  $c$  is the velocity of light,  $\mathbf{p} = m\mathbf{v}$  is the electrons momentum,  $\mathbf{v}$  is the electrons velocity.

Lower local equilibrium distribution of Fermi–Dirac is required to us,  $f^{(0)} = f_{eq}(\mathbf{r}, v)$  (eq $\equiv$  equilibrium),

$$f_{eq}(\mathbf{r}, v) = \left[ 1 + \exp \frac{\mathcal{E} - \mu(\mathbf{r})}{k_B T} \right]^{-1},$$

$\mathcal{E} = mv^2/2$  is the electron energy,  $\mu$  is the chemical potential of electronic gas,  $k_B$  is the Boltzmann constant,  $T$  is the plasma temperature,  $v_T$  is the heat electron velocity,

$$v_T = \sqrt{\frac{2k_B T}{m}}, \quad k_B T = \mathcal{E}_T = \frac{mv_T^2}{2},$$

$\mathcal{E}_T$  is the heat kinetic electron energy.

In quantum plasma velocity of electrons is connected with its momentum and vector potentials of electromagnetic fields equality

$$\mathbf{v} = \frac{\mathbf{P}}{m} - \frac{e}{cm}(\mathbf{A}_1 + \mathbf{A}_2 + \cdots + \mathbf{A}_N).$$

If we introduce the dimensionless momentum of the electrons  $\mathbf{P} = \mathbf{p}/p_T$ , where  $p_T = mv_T$  is the heat electron momentum, then the preceding equation can be written as

$$\mathbf{v} = v_T \left( \mathbf{P} - \frac{e}{cp_T}(\mathbf{A}_1 + \mathbf{A}_2 + \cdots + \mathbf{A}_N) \right).$$

We need absolute distribution of Fermi–Dirac  $f_0(p)$ ,

$$\begin{aligned} f_0(p) &= \left[ 1 + \exp \frac{p^2/2m - \mu}{k_B T} \right]^{-1} = \left[ 1 + \exp \left( \frac{p^2}{p_T^2} - \alpha \right) \right]^{-1} = \\ &= \frac{1}{1 + e^{P^2 - \alpha}} = f_0(P). \end{aligned}$$

Here  $\alpha = \mu/(k_B T)$  is the chemical potential of electronic gas.

We pass to dimensionless momentum in equation (1.1) and in Wigner' integral. We obtain the equation

$$\begin{aligned} \frac{\partial f}{\partial t} + v_T \mathbf{P} \frac{\partial f}{\partial \mathbf{r}} + \frac{iev_T}{c\hbar} \sum_{j=1}^N (\mathbf{P} \mathbf{A}_j) \left[ f(\mathbf{r}, \mathbf{P} + \frac{\mathbf{q}_j}{2}, t) - f(\mathbf{r}, \mathbf{P} - \frac{\mathbf{q}_j}{2}, t) \right] - \\ - \frac{ie^2}{2mc^2\hbar} \sum_{j=1}^N \mathbf{A}_j^2 \left[ f(\mathbf{r}, \mathbf{P} + \mathbf{q}_j, t) - f(\mathbf{r}, \mathbf{P} - \mathbf{q}_j, t) \right] - \end{aligned}$$

$$-\frac{2ie^2}{2mc^2\hbar} \sum_{\substack{s,j=1 \\ j<s}}^N \mathbf{A}_s \mathbf{A}_j \left[ f(\mathbf{r}, \mathbf{P} + \frac{\mathbf{q}_s + \mathbf{q}_j}{2}, t) - f(\mathbf{r}, \mathbf{P} - \frac{\mathbf{q}_s + \mathbf{q}_j}{2}, t) \right] = 0. \quad (1.2)$$

Here  $\mathbf{q}_j$  is the dimensionless wave numbers,  $\mathbf{q}_j = \mathbf{k}_j/k_T$ ,  $k_T$  is the heat wave number,  $k_T = mv_T/\hbar$ .

For definiteness we will consider, that wave vectors  $N$  of fields are directed along an axis  $x$  and electromagnetic fields are directed along an axis  $y$ , i.e.

$$\mathbf{A}_j = A_j(x, t)(0, 1, 0), \quad A_j(x, t) = A_{0j}e^{i(k_jx - \omega_jt)},$$

$$\mathbf{k}_j = k_j(1, 0, 0), \quad \mathbf{E}_j = E_j(x, t)(0, 1, 0), \quad E_j(x, t) = E_{0j}e^{i(k_jx - \omega_jt)}.$$

Now the equation (1.2) is somewhat simplified

$$\begin{aligned} \frac{\partial f}{\partial t} + v_T P_x \frac{\partial f}{\partial x} + \frac{iev_T}{c\hbar} \sum_{j=1}^N (P_y A_j) \left[ f(x, \mathbf{P} + \frac{\mathbf{q}_j}{2}, t) - f(x, \mathbf{P} - \frac{\mathbf{q}_j}{2}, t) \right] - \\ - \frac{ie^2}{2mc^2\hbar} \sum_{j=1}^N A_j^2 \left[ f(x, \mathbf{P} + \mathbf{q}_j, t) - f(x, \mathbf{P} - \mathbf{q}_j, t) \right] - \\ - \frac{2ie^2}{2mc^2\hbar} \sum_{\substack{s,j=1 \\ j<s}}^N A_s A_j \left[ f(x, \mathbf{P} + \frac{\mathbf{q}_s + \mathbf{q}_j}{2}, t) - f(x, \mathbf{P} - \frac{\mathbf{q}_s + \mathbf{q}_j}{2}, t) \right] = 0. \quad (1.2') \end{aligned}$$

We will search the solution of equation (1.2) in the form

$$f = f_0(P) + f_1(x, \mathbf{P}, t) + f_2(x, \mathbf{P}, t). \quad (1.3)$$

Here

$$\begin{aligned} f_1(x, \mathbf{P}, t) = A_1(x, t)\varphi_1(\mathbf{P}) + A_2(x, t)\varphi_2(\mathbf{P}) + \dots + A_N(x, t)\varphi_N(\mathbf{P}) = \\ = \sum_{j=1}^N A_j(x, t)\varphi_j(\mathbf{P}), \quad (1.4) \end{aligned}$$

where

$$\begin{aligned} A_j(x, t) \sim e^{i(k_jx - \omega_jt)}, \\ f_2(x, \mathbf{P}, t) = \sum_{b=1}^N A_b^2(x, t)\psi_b(\mathbf{P}) + \sum_{\substack{s,j=1 \\ j<s}}^N A_s(x, t)A_j(x, t)\xi_{j,s}(\mathbf{P}), \quad (1.5) \end{aligned}$$

where

$$A_b(x, t) \sim e^{2i(k_b x - \omega_b t)}, A_s(x, t)A_j(x, t) \sim e^{i[(k_s + k_j)x - (\omega_s + \omega_j)t]}.$$

In equalities (1.4) and (1.5) new unknown functions are introduced

$$\varphi_j = \varphi_j(\mathbf{P}), \quad \xi_{s,j} = \xi_{s,j}(\mathbf{P}), \quad \psi_b = \psi_b(\mathbf{P}), \quad j, s, b = 1, 2, \dots, N.$$

### 3 The solution of Wigner equation in first approximation

In this equation exist  $2N$  parameters of dimension of length  $\lambda_j = v_T/\omega_j$  ( $v_T$  is the heat electron velocity) and  $l_j = 1/k_j$ . We shall believe, that on lengths  $\lambda_j$ , so and on lengths  $l_j$  energy variable of electrons under acting correspond electric field  $A_j$  is much less than heat energy of electrons  $k_B T$  ( $k_B$  is the Boltzmann constant,  $T$  is the temperature of plasma), i.e. we shall consider small parameters

$$\alpha_j = \frac{|eA_j|v_T}{ck_B T} \quad (j = 1, 2, \dots, N)$$

and

$$\beta_j = \frac{|eA_j|\omega_j}{k_j k_B T c} \quad (j = 1, 2, \dots, N).$$

If to use communication of vector potentials electromagnetic fields with strengths of corresponding electric fields, injected small parameters are expressed following equalities

$$\alpha_j = \frac{|eE_j|v_T}{\omega_j k_B T} \quad (j = 1, 2, \dots, N)$$

and

$$\beta_j = \frac{|eE_j|}{k_j k_B T} \quad (j = 1, 2, \dots, N).$$

We will work with a method consecutive approximations, considering, that

$$\alpha_j \ll 1 \quad (j = 1, 2, \dots, N)$$

and

$$\beta_j \ll 1 \quad (j = 1, 2, \dots, N).$$

In the first approximation we search the solution of Wigner equation in the form

$$f = f^{(1)} = f_0(P) + f_1, \quad (2.1)$$

where  $f_1$  is the linear combination of vector potentials (1.4).

The Wigner equation (1.2) in linear approximation on quantities vector potentials has the following form

$$\frac{\partial f}{\partial t} + v_T P_x \frac{\partial f}{\partial x} + \frac{iev_T}{c\hbar} \sum_{j=1}^N (P_y A_j) \left[ f(x, \mathbf{P} + \frac{\mathbf{q}_j}{2}, t) - f(x, \mathbf{P} - \frac{\mathbf{q}_j}{2}, t) \right] = 0. \quad (2.2)$$

By means of a method of small parameter, we will substitute in the first two members of equation (2.2)  $f = f_1$ , and in the third member of equation  $f = f_0$ . We receive the following equation

$$\frac{\partial f_1}{\partial t} + v_T P_x \frac{\partial f_1}{\partial x} + \frac{iev_T}{c\hbar} \sum_{j=1}^N (P_y A_j) \left[ f_0(\mathbf{P} + \frac{\mathbf{q}_j}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}_j}{2}) \right] = 0. \quad (2.2')$$

Here

$$\begin{aligned} f_0\left(\mathbf{P} \pm \frac{\mathbf{q}_j}{2}\right) &= \left\{ 1 + \exp \left[ \left( \mathbf{P} \pm \frac{\mathbf{q}_j}{2} \right)^2 - \alpha \right] \right\}^{-1} = \\ &= \left\{ 1 + \exp \left[ \left( P_x \pm \frac{q_j}{2} \right)^2 + P_y^2 + P_z^2 - \alpha \right] \right\}^{-1} \end{aligned}$$

and the dimensionless parameters are introduced

$$q_j = \frac{k_j}{k_T}, \quad j = 1, 2, \dots, N,$$

$q_j$  is the dimensionless wave number,  $k_T = \frac{mv_T}{\hbar}$  is the heat wave number. We introduce also the dimensionless oscillation frequency of vector potential of electromagnetic field  $\mathbf{A}_j$ ,  $\Omega_j = \frac{\omega_j}{k_T v_T}$ .



At substitution (2.1) and (1.4) in the equation (2.2) we receive the following equation

$$\begin{aligned} & A_1\varphi_1(q_1P_x - \Omega_1) + \cdots + A_N\varphi_N(q_NP_x - \Omega_N) = \\ & = -\frac{ev_T}{c\hbar k_T v_T} \left\{ P_y A_1 \left[ f_0\left(\mathbf{P} + \frac{\mathbf{q}_1}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_1}{2}\right) \right] + \cdots + \right. \\ & \quad \left. + P_y A_N \left[ f_0\left(\mathbf{P} + \frac{\mathbf{q}_N}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_N}{2}\right) \right] \right\}. \end{aligned}$$

The last equation breaks up on the equations  $N$  the equations

$$(q_j P_x - \Omega_j) A_j \varphi_j = -\frac{e(P_y A_j)}{c\hbar k_T} \left[ f_0\left(\mathbf{P} + \frac{\mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j}{2}\right) \right], \quad j = 1, 2, \dots, N,$$

from which we receive

$$A_j \varphi_j = -\frac{e P_y A_j}{c\hbar k_T} \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j}{2}\right)}{q_j P_x - \Omega_j}, \quad j = 1, 2, \dots, N.$$

Thus Wigner's function in a first approximation is constructed

$$f_1 = -\frac{e}{c\hbar k_T} \sum_{j=1}^N (P_y A_j) \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j}{2}\right)}{q_j P_x - \Omega_j},$$

or

$$f_1 = -\frac{e}{c\hbar k_T} \sum_{j=1}^N (\mathbf{P} \mathbf{A}_j) \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j}{2}\right)}{\mathbf{q}_j \mathbf{P}_j - \Omega_j}. \quad (2.3)$$

## 4 The solution of Wigner equation in second approximation

In the second approximation we search for the decision of Wigner equation (1.2) in the form of (1.3), in which  $f_2$  it is defined by equality (1.5). By means of a method of small parameter we will substitute in the first two members of equation (1.2), we substitute  $f_2$  and in the third member of equation we substitute  $f_1$ . In the fourth and fifth members of equation we substitute  $f_0$ . We receive the following equation

$$\frac{\partial f_2}{\partial t} + v_T P_x \frac{\partial f_2}{\partial x} + \frac{iev_T}{c\hbar} \sum_{j=1}^N (P_y A_j) \left[ f_1\left(x, \mathbf{P} + \frac{\mathbf{q}_j}{2}, t\right) - f_1\left(x, \mathbf{P} - \frac{\mathbf{q}_j}{2}, t\right) \right] -$$

$$-\frac{ie^2}{mc^2\hbar} \sum_{\substack{s,j=1 \\ j<s}}^N A_s A_j \left[ f_0\left(\mathbf{P} + \frac{\mathbf{q}_s + \mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_s + \mathbf{q}_j}{2}\right) \right] = 0. \quad (1.2'')$$

Calculating the first two terms of equation and using the first approximation (2.3), we obtain the following equation

$$\begin{aligned} & iv_T k_T \sum_{s,j=1}^N \left[ (q_s + q_j) P_x - (\Omega_s + \Omega_j) \right] A_s A_j \xi_{s,j} = \\ & = \frac{ie^2}{c^2 m \hbar} \sum_{j,s=1}^N P_y^2 A_j A_s \left[ \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_s + \mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_s - \mathbf{q}_j}{2}\right)}{q_s(P_x + q_j/2) - \Omega_s} - \right. \\ & \quad \left. - \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_s - \mathbf{q}_j}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_s + \mathbf{q}_j}{2}\right)}{q_s(P_x - q_j/2) - \Omega_s} \right] + \\ & + \frac{ie^2}{2c^2 m \hbar} \sum_{j,s=1}^N A_j A_s \left[ f_0\left(\mathbf{P} + \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right) \right]. \quad (3.1) \end{aligned}$$

Let us find the sum of differences of the third term from the equation (3.1)

$$\begin{aligned} & -\frac{iev_T}{c\hbar} \left[ (\mathbf{P} \mathbf{A}_1) \left[ f_1\left(\mathbf{P} + \frac{\mathbf{q}_1}{2}\right) - f_1\left(\mathbf{P} - \frac{\mathbf{q}_1}{2}\right) \right] + \dots + \right. \\ & \quad \left. + \left[ f_1\left(\mathbf{P} + \frac{\mathbf{q}_N}{2}\right) - f_1\left(\mathbf{P} - \frac{\mathbf{q}_N}{2}\right) \right] \right] = \\ & = \frac{ie^2}{c^2 m \hbar} \left\{ \sum_{j,s=1}^N (\mathbf{P} \mathbf{A}_j) (\mathbf{P} \mathbf{A}_s) \left[ \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right)}{\mathbf{P} \mathbf{q}_s - \Omega_s + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] - \right. \\ & \quad - \left[ \frac{f_0\left(\mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right)}{\mathbf{P} \mathbf{q}_s - \Omega_s + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] + \\ & \quad + \left[ \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j + \mathbf{q}_s}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2}\right)}{\mathbf{P} \mathbf{q}_j - \Omega_j + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] - \\ & \quad \left. - \left[ \frac{f_0\left(\mathbf{P} + \frac{\mathbf{q}_j - \mathbf{q}_s}{2}\right) - f_0\left(\mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2}\right)}{\mathbf{P} \mathbf{q}_j - \Omega_j + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] \right\}. \end{aligned}$$

The composed equalities (1.5) are equal

$$\begin{aligned}
\mathbf{A}_j \mathbf{A}_s \xi_{j,s} &= \frac{e^2}{2c^2 p_T^2} \times \frac{(\mathbf{P} \mathbf{A}_j)(\mathbf{P} \mathbf{A}_s)}{\mathbf{P} \frac{\mathbf{q}_j + \mathbf{q}_s}{2} - \frac{\Omega_j + \Omega_s}{2}} \times \\
&\times \left\{ \left[ \frac{f_0 \left( \mathbf{P} + \frac{\mathbf{q}_j + \mathbf{q}_s}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j + \mathbf{q}_s}{2} \right)}{\mathbf{P} \mathbf{q}_s - \Omega_s + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] - \right. \\
&- \left[ \frac{f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j + \mathbf{q}_s}{2} \right)}{\mathbf{P} \mathbf{q}_s - \Omega_s + \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] + \\
&+ \left[ \frac{f_0 \left( \mathbf{P} + \frac{\mathbf{q}_j + \mathbf{q}_s}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2} \right)}{\mathbf{P} \mathbf{q}_j - \Omega_j - \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] - \\
&- \left. \left[ \frac{f_0 \left( \mathbf{P} + \frac{\mathbf{q}_j - \mathbf{q}_s}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j - \mathbf{q}_s}{2} \right)}{\mathbf{P} \mathbf{q}_j - \Omega_j - \frac{\mathbf{q}_j \mathbf{q}_s}{2}} \right] \right\} + \frac{e^2 \mathbf{A}_j \mathbf{A}_s}{2c^2 p_T^2} \times \\
&\times \frac{f_0 \left( \mathbf{P} + \frac{\mathbf{q}_s + \mathbf{q}_j}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_s + \mathbf{q}_j}{2} \right)}{\mathbf{P} \frac{\mathbf{q}_j + \mathbf{q}_s}{2} - \frac{\Omega_j + \Omega_s}{2}}, \quad s, j = 1, 2, \dots, N.
\end{aligned}$$

and

$$\begin{aligned}
\mathbf{A}_b^2 \psi_b &= \frac{e^2}{2c^2 p_T^2} \left[ (\mathbf{P} \mathbf{A}_b)^2 \frac{f_0(\mathbf{P} + \mathbf{q}_b) - f_0(\mathbf{P} - \mathbf{q}_b)}{\mathbf{P} \mathbf{q}_b - \Omega_b + \frac{\mathbf{q}_b^2}{2}} + \right. \\
&+ \left. \frac{\mathbf{A}_b^2}{2} (f_0(\mathbf{P} + \mathbf{q}_b) - f_0(\mathbf{P} - \mathbf{q}_b)) \right] \frac{1}{\mathbf{P} \mathbf{q}_b - \Omega_b} \quad b = 1, 2, \dots, N \quad (3.2)
\end{aligned}$$

Thus, the decision of Wigner equation is constructed and in the second approximation. It is defined by equalities (1.3)–(1.5), in which functions  $\xi_{s,j}(s, j = 1, 2, \dots, N)$  and  $\psi_b(b = 1, 2, \dots, N)$  are defined equality (3.2).

## 5 The electric current in quantum plasma

The density of electric current according to his definition is equal

$$\mathbf{j} = e \int f \mathbf{v} \frac{2d^3p}{(2\pi\hbar)^3}.$$

In work [10] it is shown, that in zeroth approximation electric current in quantum plasmas equals to zero

$$\mathbf{j}^{(0)} = e \int f_0(P) \mathbf{v} \frac{2d^3p}{(2\pi\hbar)^3} = 0.$$

Therefore the density of electric current in quantum plasmas is equal

$$\mathbf{j} = \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int (f_1 + f_2) \left( \mathbf{P} - \frac{e(\mathbf{A}_1 + \mathbf{A}_2 + \dots + \mathbf{A}_N)}{mcv_T} \right) d^3P. \quad (4.1)$$

Equality (4.1) can be presented in the form

$$\mathbf{j} = \mathbf{j}^{\text{linear}} + \mathbf{j}^{\text{quadr}}.$$

Here

$$\mathbf{j}^{\text{linear}} = \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int f_1 \mathbf{P} d^3P, \quad (4.2)$$

$$\mathbf{j}^{\text{quadr}} = \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int \left[ f_2 \mathbf{P} - \frac{e(\mathbf{A}_1 + \mathbf{A}_2 + \dots + \mathbf{A}_N)}{cp_T} f_1 \right] d^3P. \quad (4.3)$$

Electric current in quantum plasma is the sum two composed, linear and square. Linear composed there is density of the current directed along the vector capacity of the electromagnetic field (i.e. along vector of tension of the field). It consists of the members proportional to the first degree of vector potentials. Square composed there is density of current, orthogonal current of linear density. It is directed along wave vector. Square composed consists of the members proportional to square of vector potentials of current and their work.

Let us present linear part of density of current (4.2) in an explicit form

$$\mathbf{j}^{\text{linear}} = -\frac{2ep_T^3 v_T}{(2\pi\hbar)^3 c \hbar k_T} \sum_{j=1}^N \int \mathbf{P} (\mathbf{P} \mathbf{A})_j \frac{f_0 \left( \mathbf{P} + \frac{\mathbf{q}_j}{2} \right) - f_0 \left( \mathbf{P} - \frac{\mathbf{q}_j}{2} \right)}{\mathbf{P} \mathbf{q}_j - \Omega_j} d^3P.$$

This vector expression has one nonzero to component

$$\mathbf{j}^{\text{linear}} = j_y(0, 1, 0),$$

where

$$j_y = -\frac{2ep_T^3}{(2\pi\hbar)^3 cm} \sum_{j=1}^N A_j \times \\ \times \int \frac{f_0\left(P_x + \frac{q_j}{2}, P_y, P_z\right) - f_0\left(P_x - \frac{q_j}{2}, P_y, P_z\right)}{q_j P_x - \Omega_j} P_y^2 d^3 P. \quad (4.4)$$

Here designation is entered

$$f_0\left(P_x \pm \frac{q_j}{2}, P_y, P_z\right) = \frac{1}{1 + \exp\left[\left(P_x \pm \frac{q_j}{2}\right)^2 + P_y^2 + P_z^2 - \alpha\right]}.$$

Let us use further shorter designation

$$f_0\left(P_x \pm \frac{q_j}{2}\right) \equiv f_0\left(P_x \pm \frac{q_j}{2}, P_y, P_z\right).$$

Let us find numerical density the concentration of particles of plasma answering to the distribution of Fermi–Dirac

$$N_0 = \int f_0(P) \frac{2d^3 p}{(2\pi\hbar)^3} = \frac{8\pi p_T^3}{(2\pi\hbar)^3} \int_0^\infty \frac{e^{\alpha-P^2} P^2 dP}{1 + e^{\alpha-P^2}} = \frac{k_T^3}{2\pi^2} l_0(\alpha),$$

where

$$l_0(\alpha) = \int_0^\infty \ln(1 + e^{\alpha-\tau^2}) d\tau.$$

Let us enter a change of variables in (4.2) and we will enter plasma (Langmuir) frequency

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}.$$

Then we use communication between the numerical density of particles plasmas (concentration), thermal wave number of electrons and them chemical potential

$$N_0 = \frac{1}{2\pi^2} k_T^3 l_0(\alpha).$$

As a result we receive that expression for current (4.4) is equal

$$j_y = -\frac{\omega_p^2}{8\pi^2 c} \sum_{j=1}^N A_j \times \\ \times \int \left( \frac{1}{q_j P_x - \Omega_j - q_j^2/2} - \frac{1}{q_j P_x - \Omega_j + q_j^2/2} \right) f_0(P) P_y^2 d^3 P,$$

or

$$j_y = i \frac{\omega_p^2}{8\pi^2} \sum_{j=1}^N \frac{E_j q_j^2}{\omega_j} \int \frac{f_0(P) P_y^2 d^3 P}{(q_j P_x - \Omega_j)^2 - q_j^4/4}.$$

This expression of density of transversal current comes down to double integral

$$j_y = \frac{i\Omega_p^2 k_T v_T}{8\pi} \sum_{j=1}^N \frac{E_j q_j^2}{\Omega_j} \int_0^\infty \frac{P^4 dP}{1 + e^{P^2 - \alpha}} \int_{-1}^1 \frac{(1 - \mu^2) d\mu}{(q_j P \mu - \Omega_j)^2 - q_j^4/4}.$$

Here

$$\Omega_p = \frac{\omega_p}{k_T v_T} = \frac{\hbar \omega_p}{m v_T^2}$$

is the dimensionless plasma frequency.

## 6 The longitudinal current in quantum plasma

We should note that the integral from an addend in (4.3) is equal to zero. Therefore the longitudinal current in quantum plasma generated by  $N$  electromagnetic fields is equal

$$\mathbf{j}^{\text{quadr}} \equiv \mathbf{j}^{\text{long}} = \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int f_2 \mathbf{P} d^3 P. \quad (5.1)$$

Thus, longitudinal current is defined only by the second approach of function of distribution.

Vector equality (5.1) has only one nonzero to component  $\mathbf{j}^{\text{long}} = j_x(1, 0, 0)$ , where

$$j_x = \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int f_2 P_x d^3 P. \quad (5.2)$$

Longitudinal current (5.2) we will present the sums of composed in the form

$$j_x = \sum_{b=1}^N j_b + \sum_{\substack{j,s=1 \\ s < j}}^N j_{j,s}. \quad (5.3)$$

Here

$$j_b = A_b^2 \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int P_x \psi_b d^3 P, \quad (b = 1, 2, \dots, N), \quad (5.4)$$

$$j_{j,s} = A_j A_s \frac{2ep_T^3 v_T}{(2\pi\hbar)^3} \int P_x \xi_{j,s} d^3 P \quad (j, s = 1, 2, \dots, N). \quad (5.5)$$

From equality (5.3) follows, that the longitudinal current represents the sum of two components. Currents from the first sum  $j_b$  are generated by the corresponding vector potentials of electromagnetic fields. Their quantities are proportional to squares of these vector potentials. The second sum of currents which we name "crossed", are generated by interaction of electromagnetic fields among themselves and are proportional to product of quantities of their vector potentials.

We present the formula (5.4) in the explicit form

$$j_b = A_b^2 \frac{e^3 p_T v_T}{(2\pi\hbar)^3 c^2} \int \left[ \left( \frac{f_0(P_x + q_b) - f_0(P)}{q_b P_x - \Omega_b + q_b^2/2} - \frac{f_0(P) - f_0(P_x - q_b)}{q_b P_x - \Omega_b - q_b^2/2} \right) P_y^2 + \frac{f_0(P_x + q_b) - f_0(P_x - q_b)}{2} \right] \frac{P_x d^3 P}{q_b P_x - \Omega_b}, \quad (b = 1, 2, \dots, N). \quad (5.6)$$

Let us transform the expression facing integral in the previous formula

$$C = \frac{e^3 p_T v_T}{(2\pi\hbar)^3 c^2} A_b^2 = \frac{e^3 k_T^3 v_T}{8\pi^3 c^2 p_T^2} A_b^2.$$

We will use communication between concentration (numerical density), thermal wave number and chemical potential

$$N_0 = \frac{1}{2\pi^2} k_T^3 l_0(\alpha), \quad l_0(\alpha) = \int_0^\infty \ln(1 + e^{\alpha - \tau^2}) d\tau.$$

Then

$$C = A_b^2 \frac{2\pi^2 e^3 N_0 v_T}{8\pi^3 c^2 p_T^2 l_0(\alpha)} = A_b^2 \frac{e\omega_p^2}{16\pi^2 c^2 p_T l_0(\alpha)} =$$

$$= -E_b^2 \frac{e\omega_p^2}{16\pi^2 l_0(\alpha) p_T \omega_b} = -E_b^2 \frac{e\Omega_p^2}{16\pi^2 l_0(\alpha) p_T \Omega_b}.$$

Here dimensionless plasma frequencies are entered

$$\Omega_p = \frac{\omega_p}{k_T v_T}, \quad \Omega_j = \frac{\omega_j}{k_T v_T} \quad (j = 1, 2, \dots, N).$$

We introduce the longitudinal-transversal conductivity  $\sigma_{l,tr}$ ,

$$\sigma_{l,tr} = \frac{e\hbar}{p_T^2} \left( \frac{\hbar\omega_p}{mv_T^2} \right)^2 = \frac{e}{k_T p_T} \left( \frac{\omega_p}{k_T v_T} \right)^2 = \frac{e\Omega_p^2}{p_T k_T}.$$

Then we have

$$C = -\frac{E_b^2 \sigma_{l,tr} k_b}{16\pi^2 l_0(\alpha) \Omega_b^2 q_b}, \quad (b = 1, 2, \dots, N).$$

Now the formula (5.6) can be presented as

$$j_b = J_b \sigma_{l,tr} k_b E_b^2. \quad (5.7)$$

In (5.7)  $J_b$  is dimensionless current densities,

$$J_b = -\frac{1}{16\pi^2 l_0(\alpha) q_b \Omega_b} \int \left[ \left( \frac{f_0(P_x + q_b) - f_0(P)}{q_b P_x - \Omega_b + q_b^2/2} - \frac{f_0(P) - f_0(P_x - q_b)}{q_b P_x - \Omega_b - q_b^2/2} \right) P_y^2 + \frac{f_0(P_x + q_b) - f_0(P_x - q_b)}{2} \right] \frac{P_x d^3 P}{q_b P_x - \Omega_b}. \quad (5.8)$$

In (5.8) we will reduce integral to one-dimensional. For this purpose it will be necessary for us following equalities. Let us calculate internal integrals in  $(P_y, P_z)$  passing to polar coordinates

$$\begin{aligned} \int f_0(P_x \pm q_b, P_y, P_z) P_y^2 dP_y dP_z &= \int_0^{2\pi} \int_0^\infty \frac{\cos^2 \varphi \rho^3 d\varphi d\rho}{1 + e^{(P_x \pm q_b)^2 + \rho^2 - \alpha}} = \\ &= \pi \int_0^\infty \frac{\rho^3 d\rho}{1 + e^{(P_x \pm q_b)^2 + \rho^2 - \alpha}} = \pi \int_0^\infty \rho \ln(1 + e^{-(P_x \pm q_b)^2 - \rho^2 + \alpha}) d\rho, \end{aligned}$$

where

$$\rho = \sqrt{P_y^2 + P_z^2}.$$



Similarly

$$\begin{aligned}
\int f_0(P)P_y^2 dP_y dP_z &= \int_0^{2\pi} \int_0^\infty \frac{\cos^2 \varphi \rho^3 d\varphi d\rho}{1 + e^{P_x^2 + \rho^2 - \alpha}} = \\
&= \pi \int_0^\infty \rho \ln(1 + e^{-P_x^2 - \rho^2 + \alpha}) d\rho, \\
\int f_0(P \pm q_b) dP_y dP_z &= \int_0^{2\pi} \int_0^\infty \frac{\rho d\varphi d\rho}{1 + e^{(P_x \pm q_b)^2 + \rho^2 - \alpha}} = \\
&= 2\pi \int_0^\infty \frac{e^{\alpha - (P_x \pm q_b)^2 - \rho^2}}{1 + e^{\alpha - (P_x \pm q_b)^2 - \rho^2}} \rho d\rho = \pi \ln(1 + e^{\alpha - (P_x \pm q_b)^2}).
\end{aligned}$$

Therefore

$$\int_{-\infty}^\infty \int_{-\infty}^\infty f_0(P) dP_y dP_z = \pi \ln(1 + e^{\alpha - P_x^2}).$$

Let us introduce the following notation

$$\begin{aligned}
l(P_x \pm q) &= \int_0^\infty \rho \ln(1 + e^{-(P_x \pm q)^2 - \rho^2 + \alpha}) d\rho, \\
l(P_x) &= \int_0^\infty \rho \ln(1 + e^{-P_x^2 - \rho^2 + \alpha}) d\rho.
\end{aligned}$$

The integral from an addend from (5.8) is equal

$$\begin{aligned}
&\frac{1}{2} \int \frac{f_0(P_x + q_b) - f_0(P_x - q_b)}{q_b P_x - \Omega_b} P_x d^3 P = \\
&= \frac{\Omega_b}{2q_b} \int \frac{f_0(P_x + q_b) - f_0(P_x - q_b)}{q_b P_x - \Omega_b} d^3 P = \\
&= \frac{\pi \Omega_b}{2q_b} \int_{-\infty}^\infty \ln \frac{1 + e^{\alpha - (\tau + q_b)^2}}{1 + e^{\alpha - (\tau - q_b)^2}} \frac{d\tau}{q_b \tau - \Omega_b} = \\
&= \frac{\pi \Omega_b}{2q_b} \int_{-\infty}^\infty \ln(1 + e^{\alpha - \tau^2}) \left( \frac{1}{q_b \tau - \Omega_b - q_b^2} - \frac{1}{q_b \tau - \Omega_b + q_b^2} \right) d\tau =
\end{aligned}$$

$$= \pi q_b \Omega_b \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q_b \tau - \Omega_b)^2 - q_b^4} = \frac{\pi \Omega_b}{q_b} \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(\tau - \Omega_b/q_b)^2 - q_b^2}.$$

Let us calculate integral from the first item. We have

$$\begin{aligned} & \int \left( \frac{f_0(P_x + q_b) - f_0(P)}{q_b P_x - \Omega_b + q_b^2/2} - \frac{f_0(P) - f_0(P_x - q_b)}{q_b P_x - \Omega_b - q_b^2/2} \right) \frac{P_y^2 P_x d^3 P}{q_b P_x - \Omega_b} = \\ & = \pi \int_{-\infty}^{\infty} \left( \frac{L(P_x + q_b, P_x)}{q_b P_x - \Omega_b + q_b^2/2} + \frac{L(P_x - q_b, P_x)}{q_b P_x - \Omega_b - q_b^2/2} \right) \frac{P_x dP_x}{q_b P_x - \Omega_b}. \end{aligned}$$

Here

$$L(P_x \pm q_b, P_x) = l(P_x \pm q_b) - l(P_x) = \int_0^{\infty} \rho \ln \frac{1 + e^{\alpha - (P_x \pm q_b)^2 - \rho^2}}{1 + e^{\alpha - P_x^2 - \rho^2}} d\rho.$$

We will transform the considered integral as follows

$$\begin{aligned} & \int_{-\infty}^{\infty} \left( \frac{L(\tau + q_b, \tau)}{q_b \tau - \Omega_b + q_b^2/2} + \frac{L(\tau - q_b, \tau)}{q_b \tau - \Omega_b - q_b^2/2} \right) \frac{\tau d\tau}{q_b \tau - \Omega_b} = \\ & = \int_{-\infty}^{\infty} \left( \frac{\tau - q_b/2}{q_b \tau - \Omega_b - q_b^2/2} - \frac{\tau + q_b/2, \tau}{q_b \tau - \Omega_b + q_b^2/2} \right) \frac{L(\tau + q_b/2, \tau - q_b/2)}{q_b \tau - \Omega_b} d\tau = \\ & = q_b \Omega_b \int_{-\infty}^{\infty} \frac{L(\tau + q_b/2, \tau - q_b/2) d\tau}{(q_b \tau - \Omega_b)[(q_b \tau - \Omega_b)^2 - q_b^4/4]} d\tau. \end{aligned}$$

The finally dimensionless current density is equal

$$\begin{aligned} J_b = & -\frac{1}{16\pi l_0(\alpha) \Omega_b} \int_{-\infty}^{\infty} \left[ \frac{L(\tau + q_b/2, \tau - q_b/2)}{(q_b \tau - \Omega_b)(q_b \tau - \Omega_b)^2 - q_b^4/4} + \right. \\ & \left. + \frac{\ln(1 + e^{\alpha - \tau^2})}{(q_b \tau - \Omega_b)^2 - q_b^4} \right] d\tau. \end{aligned}$$

## 7 The crossed current

We rewrite the formula for calculation of crossed currents in the explicit form

$$\begin{aligned}
\sum_{\substack{s,j=1 \\ j < s}}^N j_{j,s} &= \frac{e^3 p_T^3 v_T}{(2\pi\hbar)^3 c^2 p_T^2} \sum_{\substack{s,j=1 \\ j < s}}^N A_j A_s \int \left[ \frac{f_0(P_x + q^+) - f_0(P_x - q^-)}{q_j P_x - \Omega_j + q_j q_s / 2} - \right. \\
&\quad - \frac{f_0(P_x + q^-) - f_0(P_x - q^+)}{q_j P_x - \Omega_j - q_j q_s / 2} + \frac{f_0(P_x + q^+) - f_0(P_x + q^-)}{q_s P_x - \Omega_s + q_j q_s / 2} \\
&\quad \left. - \frac{f_0(P_x - q^-) - f_0(P_x - q^+)}{q_s P_x - \Omega_s - q_j q_s / 2} \right] \frac{P_y^2 P_x d^3 P}{q P_x - \Omega} + \\
&\quad + \frac{e^3 p_T^3 v_T}{(2\pi\hbar)^3 c^2 p_T^2} \sum_{\substack{s,j=1 \\ j < s}}^N A_j A_s \int \frac{f_0(P_x + q) - f_0(P_x - q)}{q P_x - \Omega} P_x d^3 P.
\end{aligned}$$

We rewrite this equation

$$\sum_{\substack{s,j=1 \\ j < s}}^N j_{j,s} = \frac{e^3 p_T v_T}{(2\pi\hbar)^3 c^2} \sum_{\substack{s,j=1 \\ j < s}}^N A_j A_s (J_1 - J_2 + J_3 - J_4 + J_5). \quad (6.1)$$

Here

$$\begin{aligned}
J_1 &= \int \frac{f_0(P_x + q^+) - f_0(P_x - q^-)}{q_j P_x - \Omega_j + q_j q_s / 2} \frac{P_y^2 P_x d^3 P}{q P_x - \Omega}, \\
J_2 &= \int \frac{f_0(P_x + q^-) - f_0(P_x - q^+)}{q_j P_x - \Omega_j - q_j q_s / 2} \frac{P_y^2 P_x d^3 P}{q P_x - \Omega}, \\
J_3 &= \int \frac{f_0(P_x + q^+) - f_0(P_x + q^-)}{q_s P_x - \Omega_s + q_j q_s / 2} \frac{P_y^2 P_x d^3 P}{q P_x - \Omega}, \\
J_4 &= \int \frac{f_0(P_x - q^-) - f_0(P_x - q^+)}{q_s P_x - \Omega_s - q_j q_s / 2} \frac{P_y^2 P_x d^3 P}{q P_x - \Omega}, \\
J_5 &= \int \frac{f_0(P_x + q) - f_0(P_x - q)}{q P_x - \Omega} P_x d^3 P.
\end{aligned}$$

Here

$$q = q^+ = \frac{q_j + q_s}{2}, \quad q^- = \frac{q_j - q_s}{2}, \quad \Omega = \Omega^+ = \frac{\Omega_j + \Omega_s}{2}.$$

For this purpose it will be necessary for us following equalities. Let us calculate internal integrals in  $(P_y, P_z)$  passing to polar coordinates

$$\begin{aligned} \int f_0(P_x \pm q^\pm, P_y, P_z) P_y^2 dP_y dP_z &= \int_0^{2\pi} \int_0^\infty \frac{\cos^2 \varphi \rho^3 d\varphi d\rho}{1 + e^{(P_x \pm q^\pm)^2 + \rho^2 - \alpha}} = \\ &= \pi \int_0^\infty \frac{\rho^3 d\rho}{1 + e^{(P_x \pm q^\pm)^2 + \rho^2 - \alpha}} = \pi \int_0^\infty \rho \ln(1 + e^{-(P_x \pm q^\pm)^2 - \rho^2 + \alpha}) d\rho, \end{aligned}$$

where

$$\rho = \sqrt{P_y^2 + P_z^2}.$$

Similarly

$$\begin{aligned} \int f_0(P) P_y^2 dP_y dP_z &= \\ &= \int_0^{2\pi} \int_0^\infty \frac{\cos^2 \varphi \rho^3 d\varphi d\rho}{1 + e^{P_x^2 + \rho^2 - \alpha}} = \pi \int_0^\infty \rho \ln(1 + e^{-P_x^2 - \rho^2 + \alpha}) d\rho, \\ \int f_0(P \pm q) dP_y dP_z &= \int_0^{2\pi} \int_0^\infty \frac{\rho d\varphi d\rho}{1 + e^{(P_x \pm q)^2 + \rho^2 - \alpha}} = \\ &= 2\pi \int_0^\infty \frac{e^{\alpha - (P_x \pm q)^2 - \rho^2}}{1 + e^{\alpha - (P_x \pm q)^2 - \rho^2}} \rho d\rho = \pi \ln(1 + e^{\alpha - (P_x \pm q)^2}). \end{aligned}$$

Let us introduce the following notation

$$\begin{aligned} l(P_x \pm q) &= \int_0^\infty \rho \ln(1 + e^{-(P_x \pm q)^2 - \rho^2 + \alpha}) d\rho, \\ l(P_x) &= \int_0^\infty \rho \ln(1 + e^{-P_x^2 - \rho^2 + \alpha}) d\rho. \end{aligned}$$

After long transformations  $J_1, \dots, J_5$  can be reduced to one-dimensional

$$J_1 = \pi \int_{-\infty}^\infty \frac{l(\tau + q^+) - l(\tau - q^-)}{q_j \tau - \Omega_j + q_j q_s / 2} \cdot \frac{\tau d\tau}{q\tau - \Omega},$$

$$\begin{aligned}
J_2 &= \pi \int_{-\infty}^{\infty} \frac{l(\tau + q^-) - l(\tau - q^+)}{q_j \tau - \Omega_j - q_j q_s / 2} \cdot \frac{\tau d\tau}{q\tau - \Omega}, \\
J_3 &= \pi \int_{-\infty}^{\infty} \frac{l(\tau + q^+) - l(\tau + q^-)}{q_s \tau - \Omega_s + q_j q_s / 2} \cdot \frac{\tau d\tau}{q\tau - \Omega}, \\
J_4 &= \pi \int_{-\infty}^{\infty} \frac{l(\tau - q^-) - l(\tau - q^+)}{q_s \tau - \Omega_s - q_j q_s / 2} \cdot \frac{\tau d\tau}{q\tau - \Omega}, \\
J_5 &= 2\pi q^+ \Omega \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - \tau^2}) d\tau}{(q^+ \tau - \Omega)^2 - q^{+4}}.
\end{aligned}$$

Numerators are respectively equal in integrals  $J_1, \dots, J_4$

$$\begin{aligned}
l(\tau + q^+) - l(\tau - q^-) &= \int_0^{\infty} \rho \ln \frac{1 + e^{\alpha - (\tau + q^+)^2 - \rho^2}}{1 + e^{\alpha - (\tau - q^-)^2 - \rho^2}} d\rho, \\
l(\tau + q^-) - l(\tau - q^+) &= \int_0^{\infty} \rho \ln \frac{1 + e^{\alpha - (\tau + q^-)^2 - \rho^2}}{1 + e^{\alpha - (\tau - q^+)^2 - \rho^2}} d\rho, \\
l(\tau + q^+) - l(\tau + q^-) &= \int_0^{\infty} \rho \ln \frac{1 + e^{\alpha - (\tau + q^+)^2 - \rho^2}}{1 + e^{\alpha - (\tau + q^-)^2 - \rho^2}} d\rho, \\
l(\tau - q^-) - l(\tau - q^+) &= \int_0^{\infty} \rho \ln \frac{1 + e^{\alpha - (\tau - q^-)^2 - \rho^2}}{1 + e^{\alpha - (\tau - q^+)^2 - \rho^2}} d\rho.
\end{aligned}$$

Let us enter into a formula (6.1) a plasma frequency. Now the formula (6.1) can be presented as

$$\sum_{\substack{s,j=1 \\ j < s}}^N j_{j,s} = \frac{e\omega_p^2}{16\pi^2 c^2 p_T l_0(\alpha)} \sum_{\substack{s,j=1 \\ j < s}}^N A_j A_s (J_1 - J_2 + J_3 - J_4 + J_5). \quad (6.2)$$

In a formula (6.2) we will pass from sizes of vector potentials electromagnetic fields to strengths of electric fields

$$\sum_{\substack{s,j=1 \\ j < s}}^N j_{j,s} =$$

$$= -\frac{e\Omega_p^2}{16\pi^2 l_0(\alpha)\Omega_j\Omega_s p_T} \sum_{\substack{s,j=1 \\ j<s}}^N E_j E_s (J_1 - J_2 + J_3 - J_4 + J_5). \quad (6.3)$$

We introduce again the longitudinal-transversal conductivity  $\sigma_{l,tr}$

$$\sigma_{l,tr} = \frac{e\hbar}{p_T^2} \left( \frac{\hbar\omega_p}{mv_T^2} \right)^2 = \frac{e}{k_T p_T} \left( \frac{\omega_p}{k_T v_T} \right)^2 = \frac{e\Omega_p^2}{p_T k_T}.$$

Now the formula (6.3) can be presented

$$\begin{aligned} & \sum_{\substack{s,j=1 \\ j<s}}^N j_{j,s} = \\ & = -\frac{\sigma_{l,tr}}{16\pi^2 l_0(\alpha)} \sum_{\substack{s,j=1 \\ j<s}}^N \frac{E_j E_s (k_j + k_s)}{\Omega_j \Omega_s (q_j + q_s)} (J_1 - J_2 + J_3 - J_4 + J_5). \end{aligned} \quad (6.4)$$

We will write a formula (6.4) in the form

$$\sum_{\substack{s,j=1 \\ j<s}}^N j_{j,s} = \sum_{\substack{s,j=1 \\ j<s}}^N J_{j,s} \sigma_{l,tr} E_j E_s (k_j + k_s). \quad (6.5)$$

In a formula (6.5)  $J_{j,s}$  is the dimensionless part of density cross current

$$\begin{aligned} & \sum_{\substack{s,j=1 \\ j<s}}^N J_{j,s} = \\ & = -\frac{1}{16\pi^2 l_0(\alpha)} \sum_{\substack{s,j=1 \\ j<s}}^N \frac{1}{\Omega_j \Omega_s (q_j + q_s)} (J_1 - J_2 + J_3 - J_4 + J_5). \end{aligned}$$

Thus, a longitudinal part of current is equal

$$j_x = \sigma_{l,tr} \left[ \sum_{b=1}^N E_b^2 k_b J_b + \sum_{\substack{s,j=1 \\ j<s}}^N E_j E_s (k_j + k_s) J_{j,s} \right]. \quad (6.6)$$

If to enter transversal fields

$$\mathbf{E}_j^{\text{tr}} = \mathbf{E}_j - \frac{\mathbf{k}_j(\mathbf{E}_j \mathbf{k}_j)}{k_j^2} = \mathbf{E}_j - \frac{\mathbf{q}_j(\mathbf{E}_j \mathbf{q}_j)}{q_j^2}, \quad (j = 1, 2, \dots, N).$$

then equality (6.6) can be written down in an invariantly form

$$\mathbf{j}^{\text{long}} = \sigma_{l,tr} \left[ \sum_{b=1}^N (\mathbf{E}_b^{tr})^2 \mathbf{k}_b J_b + \sum_{\substack{s,j=1 \\ j < s}}^N \mathbf{E}_j^{tr} \mathbf{E}_s^{tr} (\mathbf{k}_j + \mathbf{k}_s) J_{j,s} \right].$$

## 8 Small values of wave numbers

In case of small values of wave numbers of size of the currents proportional to squares of strengths of electric fields, are actually calculated in work [20]

$$\begin{aligned} j_m &= -\frac{e}{8\pi\omega_m} \left( \frac{\omega_p}{\omega_m} \right)^2 k_m E_m^2 = -\frac{e\Omega_p^2}{8\pi\omega_m\Omega_m^2} k_m E_m^2 = \\ &= -\frac{\sigma_{l,tr}}{8\pi\Omega_m^3} k_m E_m^2, \quad q_m \rightarrow 0, \quad (m = 1, 2, \dots, N). \end{aligned}$$

Now we will consider the size of a cross current at small values of wave numbers. We get the equation

$$f_0(P_x + q) = f_0(P) - g(P)2P_x q + \dots, \quad (q \rightarrow 0).$$

Here

$$g(P) = \frac{e^{P^2 - \alpha}}{(1 + e^{P^2 - \alpha})^2} = \frac{e^{\alpha - P^2}}{(1 + e^{\alpha - P^2})^2}.$$

Let us notice that

$$f_0(P + q) - f_0(P - q) = -4g(P)P_x q + \dots, \quad (q \rightarrow 0).$$

Therefore, density of the cross current is equal

$$\begin{aligned} &\sum_{\substack{s,j=1 \\ j < s}}^N j_{j,s} = \\ &= -\frac{\sigma_{l,tr}}{4\pi^2 l_0(\alpha)} \sum_{\substack{s,j=1 \\ j < s}}^N E_j E_s (k_j + k_s) \frac{q}{(q_j + q_s)\Omega_j\Omega_s\Omega} \int g(P) P_x^2 d^3 P. \end{aligned}$$

The integral is equal

$$\int g(P) P_x^2 d^3 P = \frac{4\pi}{3} \int_0^\infty \frac{P^4 e^{P^2-\alpha} dP}{(1 + e^{P^2-\alpha})^2} = \pi l_0(\alpha).$$

Therefore, the density of crossed currents equals

$$\sum_{\substack{j,s=1 \\ j<s}}^N j_{j,s} = -\frac{\sigma_{l,tr}}{4\pi^2 l_0(\alpha)} \sum_{\substack{j,s=1 \\ j<s}}^N E_j E_s (k_j + k_s) \frac{q}{(q_j + q_s) \Omega_j \Omega_s \Omega}.$$

We receive at small values wave numbers for density of longitudinal current

$$j_x = -\frac{\sigma_{l,tr}}{8\pi} \left[ \sum_{b=1}^N E_b^2 \frac{k_1}{\Omega_b^3} + 2 \sum_{\substack{s,j=1 \\ j<s}}^N E_j E_s \frac{k_j + k_s}{\Omega_j \Omega_s (\Omega_j + \Omega_s)} \right].$$

We write this formula in a vector form

$$\mathbf{j}^{\text{long}} = -\frac{\sigma_{l,tr}}{8\pi} \left[ \sum_{b=1}^N (\mathbf{E}_b^{tr})^2 \mathbf{k}_b \frac{b}{\Omega_b^3} + 2 \sum_{\substack{s,j=1 \\ j<s}}^N \mathbf{E}_j^{tr} \mathbf{E}_s^{tr} (\mathbf{k}_j + \mathbf{k}_s) \frac{1}{\Omega_j \Omega_s (\Omega_j + \Omega_s)} \right].$$

This formula in accuracy matches the corresponding formula from work [22]. It means that at small values of wave numbers the size of density of longitudinal current in classical and quantum plasma matches.

REMARK. At calculation of the singular integrals entering dimensionless parts of density of longitudinal current it is necessary to use the known rule of Landau.

## 9 Conclusions

This article is the continuous of our works [21]–[26]. In the present work the following problem is solved: in quantum plasma with arbitrary degree of degeneration of electronic gas, propagate  $N$  electromagnetic waves with collinear wave vectors. The Wigner equation dares by a method consecutive, approximately considering as small parameteres of one order of quantities of



intensities corresponding electric fields. Square-law decomposition of function of distribution is used.

It has appeared, that the account of nonlinearity of electromagnetic fields finds out generating of an electric current, orthogonal electric field to a direction (i.e. to a direction of a known classical transversal electric current). The quantities of transversal and longitudinal electric currents are found. The case of small values of wave numbers is considered. It turned out, that the quantities of a longitudinal current in classical and quantum plasma coincides.

Further authors purpose to consider problems about fluctuations of plasma and about skin-effect with use square-law on potential of decomposition of function of distribution.

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